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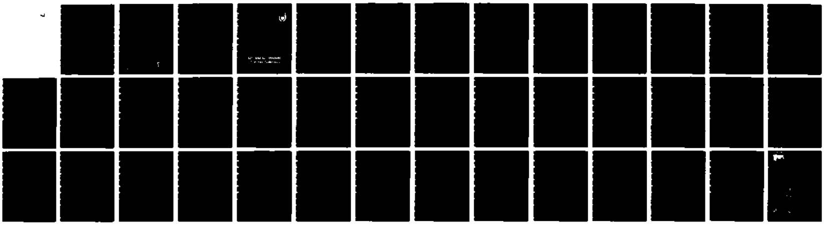

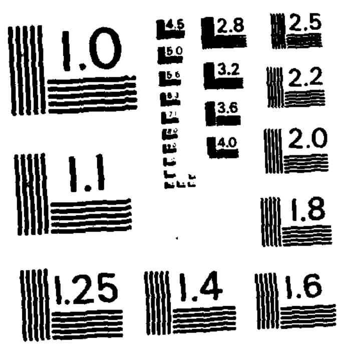

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Paper No. 83-360. Satellite Orbit Theory for a Small Computer.* R.I. Abbot, Dept. of Earth and Planetary Sciences, MIT; P. Cefola, Draper Laboratory; S.F. Tse, Dept. of Aeronautics and Astronautics, MIT; Cambridge, Mass.

A computer program has been put onto an LSI-11 microprocessor with 64KB of memory which can provide accurate ephemerides for GPS (Global Positioning System) satellites.

The satellite dynamics include averaged orbital element rates due to J2, tesseral resonances, solar radiation pressure and third body perturbations from both the Moon and the Sun. These rates are first integrated up to and across a satellite pass of interest, and a two point Hermitian interpolating polynomial is established for each mean element. Short periodic Fourier coefficients due to J2 and the Moon and Sun are next computed, and three point Lagrangian interpolating polynomials are established for them across the pass. Both sets of interpolating polynomials are finally used to provide osculating orbital elements at arbitrary times during the pass.

The modular form of the computer program and the sequential structure of the theory mean that no large portion of the program needs to exist in core at once. The use of overlays, then, allows the small computer to handle the large program.

The computation of the interpolating coefficients for orbital element prediction during a satellite pass seven days beyond the epoch time takes about 100 seconds in real time. Once the interpolating coefficients are computed the state vector of the satellite at some desired time during the satellite pass can be obtained in less than 1 second of real time. Comparisons with comparable runs with the Draper Lab GTDS R&D program have shown discrepancies in position less than 20 meters.

The computer program includes an analytical Lunar/Solar ephemeris so it is self-contained except for input mean orbital elements. Partial derivatives have been implemented which will give the capability to fit observations of the satellites and to consequently obtain the necessary mean elements.

The program can be modified quite easily to handle synchronous satellites by modifying the subroutine modules for tesseral resonant perturbations and lunar-solar short-periodics. With the present overlay scheme, considerable expansion of the program is possible to obtain more accuracy and versatility.

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INTRODUCTION

The motivation for this work came as a result of desiring accurate ephemerides for GPS (Global Positioning System) satellites which could be computed on an LSI-11 microprocessor with 64 KB of memory. The satellite ephemerides are required for MITES data reduction which is done on the LSI computer. MITES is a system of miniature radio interferometer terminals for the measurement of baseline vectors on the ground by means of interferometric observations of GPS satellites (Ref. 1).

A model was needed which would run reasonably fast and of course would fit into the small computer. Although not attained yet, our eventual accuracy goals are at the one part per million level or better.

The needs of the MITES project seemed to best be satisfied with the GTDS semianalytical orbit model used on the AMDAHL computer at Draper Labs. The advantage of using this type of model over present analytical (General Perturbations) theories is the generality of the force models which can be incorporated and the ease with which these models may be included or replaced. The latter is important for tailoring the full scale GTDS version to the LSI.

The tailoring study will be presented in more detail following a general description of the model. Following that the various force models will be referenced and discussed. Then a discussion of the computer program is given followed by the results of comparison with more complete orbit theories.

GENERAL DESCRIPTION OF THE MODEL

We begin by giving a general overview of the model.

Instead of the more traditional Keplerian elements the model uses equinoctial elements which are expressed in terms of the Keplerian elements viz

$$a = a$$

$$h = e \sin(\omega + \Omega)$$

$$k = e \cos(\omega + \Omega)$$

$$p = \tan(i/2)\sin(\Omega)$$

$$q = \tan(i/2)\cos(\Omega)$$

$$\lambda = M + \omega + \Omega$$

where a = semimajor axis, e = eccentricity, i = inclination, M = mean anomaly, ω = argument of pericenter, and Ω = longitude of ascending node.

These elements vary secularly with short periodic effects superimposed. The osculating value of an element at some time is therefore the sum of the mean value of the element and the short periodic contributions. One method of computing the osculating values is with a three part model which includes:

1) the computation of the mean values of the elements;

2) the computation of the coefficients for the Fourier series expansions of the short periodics of the elements; and

3) the combination of the results of the two.

This is accomplished as follows:

The mean values are determined from averaged rate equations which are integrated with a fourth order Runge Kutta technique. The rate equation for each element is the sum of individual rate equations due to the various perturbations. The rate equations are integrated from a specific epoch (when initial mean elements are given) up to the beginning of a satellite pass. The typical step size of the Runge Kutta integrations is 1/2 day. The rate equations are then integrated in one step to the end of the pass. Using the resultant equinoctial elements at the beginning and end of the pass, a two point Hermitian interpolating polynomial is determined for each element. This polynomial provides the averaged value of the element at any time during the pass.

The short periodic Fourier coefficients are computed at three epochs across the pass - the beginning, middle and end. The Hermitian interpolating polynomials are used to obtain the mean elements required for the evaluation of the short periodic coefficients. A sine and cosine coefficient for each of four frequencies of λ is computed as a sum over all contributing perturbations. A three-point Lagrangian interpolating polynomial is then determined for each coefficient.

Finally, there is the construction of the osculating equinoctial elements at a desired time during the pass. This is done simply by:

a) obtaining the mean elements from the Hermitian interpolators;

b) obtaining the short periodic coefficients from the Lagrangian interpolators;

c) evaluating the Fourier series with the short periodic coefficients and the mean values of λ to obtain the short periodic perturbations in the elements; and

d) adding the mean elements to the short periodic variations to obtain the osculating values.

TAILORING OF THE MODEL FOR A SMALL COMPUTER

The complete GTDS semianalytical theory provides 10 cm to 1 meter accuracy in comparison with a precision (Cowell) numerical integration of the GPS orbit (Ref. 2). The precision integration was performed using a GTDS program as modified at Draper (Ref. 3) and included all the terms significant for GPS:

- 8 x 8 gravity field
- lunar-solar point masses
- solar radiation pressure.

Designing a tailored theory first means specifying truncations for the rate equations and for the short-periodic models. The design of the tailored theory is greatly aided by output reports, from the GTDS semianalytical program, of the mean element rates and the short periodic coefficients. Summaries of these reports are given in Tables I (for the mean element rates) and II (for the short-periodic coefficients).

In analyzing the data of Table I, the following error bounds were adopted:

$$(\Delta t)(\Delta\dot{\bar{a}}) < 20 \text{ meters}$$
$$(a)(\Delta t)(\Delta\dot{\bar{h}}) < 20 \text{ meters}$$
$$(a)(\Delta t)(\Delta\dot{\bar{k}}) < 20 \text{ meters}$$
$$(a)(\Delta t)(\Delta\dot{\bar{p}}) < 20 \text{ meters}$$
$$(a)(\Delta t)(\Delta\dot{\bar{q}}) < 20 \text{ meters}$$
$$(a)(\Delta t)(\Delta\dot{\bar{\lambda}}) < 20 \text{ meters}$$

Table I

GPS Averaged Differential Equations

Term	Rate ROM*, sec^{-1}
J_2	9×10^{-9}
Lunar-Solar Point Masses	3×10^{-10}
Tesseral Resonance	1.5×10^{-11}
Solar Radiation Pressure	1×10^{-11}
J_2^2	7×10^{-13}

*ROM = Rough Order of Magnitude

Table II

GPS Short Periodics

Term	Magnit
J_2	2 km
Lunar-Solar Point Masses	200 meters
Tesseral M-Dailies	20 meters
Tesseral Linear Combination Terms	15 meters
Solar Radiation Pressure	15 meters
J_2^2	15 cm

where a, the semimajor axis of the GPS orbit is in meters and Δt is an approximation interval in seconds. These bounds relate, in a very rudimentary way, to bounds in the radial (h and k), cross track (p and q) and along track (λ) errors. Assuming a = 26559 km and Δt = 14 days gives an element rate error bound

$$\Delta \dot{\bar{a}}_i < 6 \times 10^{-13}.$$

Based on this bound, we have currently adopted all of the averaged equation of motion models in Table I except for those for J_2^2. Higher degree zonals (J_3, J_4, etc.) were eliminated either due to the near critical inclination, the high altitude and small R_E/a (where R_E is the Earth radius) or the small eccentricity of the GPS orbit. For the tesseral resonance, it was known from a previous study (Ref. 4) that the $(C,S)_{3,2}$ terms were dominant. Therefore, the nominal tesseral resonance model should include at least the $(C,S)_{3,2}$ terms. Reference 4 and further study, which will be discussed below, also indicates that the $(C,S)_{2,2}$ and $(C,S)_{4,4}$ terms have to be included. Full recursive models for the lunar-solar point mass terms (Ref. 5) and a numerical averaging algorithm for solar radiation pressure (Ref. 6) were adopted for the testing.

Based on Table II and the 20 meter requirement, it was decided to retain the J_2 zonal short periodics, the P_2 third-body short periodics and the tesseral m-dailies. Further, it was decided not to include any Weak-Time-Dependent (WTD) (Ref. 7) corrections to the third-body short-periods. The WTD terms are budgeted at 10 meters for the GPS case. It is estimated that the P_3 third-body short periodics are 15 meters for this case.

To examine the assumptions of the tailored theory, tests were made against the GTDS precision numerical integration. The procedure is as follows:

1. generate a truth ephemeris with the precise numerical integration (Cowell) with its complete force models;

2. least squares fit the tailored semianalytical satellite thoery to the truth ephemeris;

3. generate statistics and plots for the residuals between the semianalytical ephemeris (using the adjusted intitial conditions from the least squares fit) and the truth ephemeris constructed in step 1.

The major results are given in Table III. The m-daily tesseral short periodics were not included in this testing.

AVERAGED RATES AND SHORT PERIODICS

With the exception of solar radiation pressure, the force models and development of the averaged rate equations and short periodic expansions for the perturbations have been documented in the literature. With regard to these then, we will only show a table of references (Table IV) and make a few specific comments about their implementation on the LSI. A simplified analytical model has been implemented for the solar radiation pressure averaged rate equations. As this contribution to the theory has not been documented in the literature, the mathematical development will be briefly summarized.

Two forms of the J_2 averaged rate equations were constructed for the LSI model - a truncated form and a closed form in the

Table III

Tailoring Study Verification

CASE	FIT SPAN	AVERAGED ORBIT GENERATOR				SHORT PERIODICS		ERRORS (meters)			
		Zonals	Tesseral Resonance	L-S	SRP	Zonals	L-S	Radial	Cross Track	Along Track	Total
A	14^d	J_2	No	No	No	No	No	251	2206	1163	2507
B	14^d	"	(3,2)	"	Yes	J_2*	3 freq.	209	36	533	573
C	14^d	"	(3,2) (2,2)	"	"	"	"	9.4	25.9	242	243
D	14^d	"	(3,2) (2,2) (4,4)	"	"	"	"	6.7	9.8	24.2	27
E	14^d	"	(3,2) (2,2) (4,4) small •	"	No	"	"	222	10.6	433	487

Table IV

Semianalytical Satellite Theory References

Perturbation	Averaged Orbit Generator	Short-Periodic Generator
CENTRAL BODY NONSPHERICITY		
Zonals - J_2	Ref. 8	Ref. 8
Tesseral resonances	Refs. 2 & 8	
Tesseral m-dailies		Refs. 2, 8 & 9
THIRD-BODY POINT MASS		
Single phase angle models	Ref. 5	Ref. 10 as corrected by Ref. 11

eccentricity. The forms are nearly identical except that the closed form has an additional factor of X to a power where $X = 1/\sqrt{1-h^2-k^2}$. Little is gained with regard to accuracy by using the closed form, but on the other hand little time is lost computationally. Therefore the closed form has been adopted.

Initially it was felt that due to the low eccentricity of the GPS satellites ($e < 0.009$) the Fourier expansions of the J_2 short periodic generator need include just 3 frequencies of λ. Early testing of the LSI model against the GTDS model for a GPS satellite with $e = 0.003$ seemed to indicate that this was a good assumption. In tests involving a GPS satellite with $e = 0.009$, however, discrepancies larger than expected occurred. These were eventually eliminated by including 4λ frequency terms in the Fourier expansions.

The rate equations that were constructed for the tesseral resonances were all truncated to 0th order in the eccentricity.

The tesseral m-daily model has not been fully implemented in the LSI yet and so the results which will be presented below do not contain their effect.

The computation of the averaged differential equations and short periodics for the lunar-solar perturbations require ephemerides for the Moon and Sun. For the LSI model we decided to utilize the low precision Fourier series formulae of T. C. van Flandern and K. F. Pulkkinen of the USNO for the geocentric positions of the Moon and Sun (Ref. 12). The original code is very lengthy, requiring ≈ 24KB for the Moon and ≈ 4.3KB the Sun. Therefore, for the initial implementation of the USNO lunar/solar ephemeris on the LSI, we used a truncated form of the formulae due to Belastock (Ref. 13) which deleted terms of order x 10^{-4} and x 10^{-5}. In a study similar to that which was used to prepare Table III, Belastock showed an error of about 20 meters due to this truncation. Eventually we were able to fit the non-truncated series expressions into the LSI (by breaking the subroutine for the Moon into four sections and overlaying each part against the other), and eliminated this source of error.

The following briefly reviews the development of the simplified analytical model for solar radiation pressure. We should point out that the only new aspect of this model is its development in terms of equinoctial elements.

For special perturbation integrators such as the GTDS system, the solar radiation pressure is modeled with the formula (Ref. 14).

$$\ddot{\vec{R}}_{SR} = \nu P_S R_{sun}^2 \frac{C_R A}{m} \frac{\vec{R}_{VS}}{|\vec{R}_{VS}|^3}$$

where

ν = eclipse factor,

P_S = (mean solar flux at 1 AU)/(speed of light),

R_{sun} = 1 AU,

C_R = 1 + surface reflectivity,

A = effective area,

m = mass,

$\vec{R}_{VS}$ = $\vec{r} - \vec{R}_S$ where

$\vec{r}$ = position vector of the spacecraft in inertial coordinates

$\vec{R}_S$ = position vector of the Sun in inertial coordinates.

In the following the satellite is assumed to always be in sunlight so that $\nu = 1$ and the factor $P_S R^2_{sun} \frac{C_R A}{m}$ is constant.

With

$$T \equiv \nu P_S R^2_{sun} \frac{C_R A}{m},$$

then

$$\vec{R}_{SR} = \frac{T\vec{R}_{VS}}{|\vec{R}_{VS}|^3}.$$

The geometrical situation is similar to a third body mass perturbation where the Earth-center to satellite distance is very small relative to the Earth-center to third body distance. This leads us to write the equivalent disturbing potential (Ref. 15)

$$U_{SR} = \frac{-T}{\Delta}$$

where Δ as usual is written

$$\Delta = R_S[1 + (\frac{r}{R_S})^2 - \frac{2r}{R_S}\cos\psi]^{1/2}$$

and ψ is the angle between $\vec{r}$ and $\vec{R}_S$. With the Legendre expansion (Ref. 16), we obtain the disturbing potential

$$1) \qquad U_{SR} = \frac{-T}{R_S} \sum_{n=1}^{\infty} (\frac{r}{R_S})^n P_n(\cos\psi).$$

The limitations of this equation are:

1. the surface reflectivity is constant;

2. the cross-sectional area perpendicular to the flux is constant;

3. the satellite is not eclipsed.

The third assumption means that during eclipsing periods the potential cannot strictly account for solar radiation pressure. For GPS satellites these periods occur for 4-5 weeks twice a year.

The above assumptions and the smallness of r/R_S suggests that equation 1) be written:

$$2) \quad U_{SR} = \frac{-Tr\cos\phi}{R_S^2} = \frac{-T(R_S \cdot \vec{r})}{R_S^2}$$

Four more stages of algebra lead to the desired differential equations:

1. substitution of $R_S = \alpha f + \beta g + \gamma w$ and $\vec{r} = (r\cos L)f + (r\sin L)g$ into equation 2) where α, β and γ are the direction cosines of the Sun relative to the equinoctial f, g, w reference system;

2. averaging over the mean longitude L;

3. taking partial derivatives of U_{SR} with respect to a, h, k, α and β;

4. substituting these resultant expressions into equation (3-71) of Ref. 17).

The final expressions are:

$$\frac{da}{dt} = 0$$

$$\frac{dh}{dt} = Va[\frac{B}{A}\alpha + \frac{k\gamma}{AB}(hq - kp)]$$

$$\frac{dk}{dt} = -Va[\frac{B}{A}\beta + \frac{h\gamma}{AB}(hq - kp)]$$

$$\frac{dp}{dt} = Va(\frac{C}{2AB})h\gamma$$

$$\frac{dq}{dt} = Va(\frac{C}{2AB})k\gamma$$

$$\frac{d\lambda}{dt} = Va[-\frac{2}{A}(\alpha k + \beta h) + \frac{B}{A(1+B)}(h\alpha - k\beta) + \frac{\gamma}{AB}(hq - kp)]$$

where

$$V = \frac{3T}{2R_S^2}$$

$$A = na^2,\ n = \text{mean motion}$$

$$B = \sqrt{1 - h^2 - k^2}$$

$$C = 1 + p^2 + q^2.$$

PARTIAL DERIVATIVES

The purpose of computing partial derivatives in the model is to determine from data, in a least squares sense, the mean elements at epoch. The data can be in two forms:

- observations
- satellite state vectors generated by numerical integration.

If one if fitting to numerically integrated data the general expression for the partial is

$$\left.\frac{\partial x,\ \dot{x}}{\partial(\text{mean elm.})}\right|_{t_o} = \left.\frac{\partial x,\ \dot{x}}{\partial(\text{osc. elm.})}\right|_{t} \cdot \left.\frac{\partial(\text{osc. elm.})}{\partial(\text{mean elm.})}\right|_{t} \cdot \left.\frac{\partial(\text{mean elm.})}{\partial(\text{mean elm.})}\right|_{t_o}^{t} .$$

The first matrix on the right of this equation is derived from two body equations (Ref. 7), the second matrix for our purposes is the identity matrix and the third matrix has variational equations (Ref. 7) which must be integrated via Runge Kutta at the same time that the averaged rate equations are integrated in the model.

If one is fitting to observations one has

$$\frac{\partial(\text{obs.})}{\partial(\text{mean elm.})_{t_o}} = \frac{\partial(\text{obs.})}{\partial x,\ \dot{x}} \cdot \frac{\partial x,\ \dot{x}}{\partial(\text{mean elm.})_{t_o}} .$$

THE COMPUTER PROGRAM

The LSI-11 has 64 KB of main memory. Approximately 8 KB of this is occupied by system routines leaving 56 KB for the program to be executed. Fortunately, due to the overlay capability of the LSI, this is not the upper limit to the size of a program. A program using overlays can be considerably larger than would normally fit into the central memory since portions of the program reside on disk.

The basic requirement for defining an overlay structure is that only relatively small parts of the program be required for execution at any one time. An attractive feature of the semi-analytical model for implementation on a small computer is the sequential nature of its computations which makes it quite suitable for overlaying. Figures 1, 2, 3 and 4 show the subroutine structure of the program. Each diagram is a complete path of computations requiring only the end results of the previous diagrams. Within each diagram, as well, a distinct sequential structure is clear.

On the LSI the overlay structure which we set up consists of a root and three overlay regions. The root contains those parts of the program that must always reside in memory which in our case is the main program and a few global subroutines. Each of the three overlay regions contain a number of segments. Each segment consists of a group of subroutines which are independent of subroutines in other segments of that region, i.e. from one segment of the region one can't call a routine in another segment of the same region. This is important since only one segment

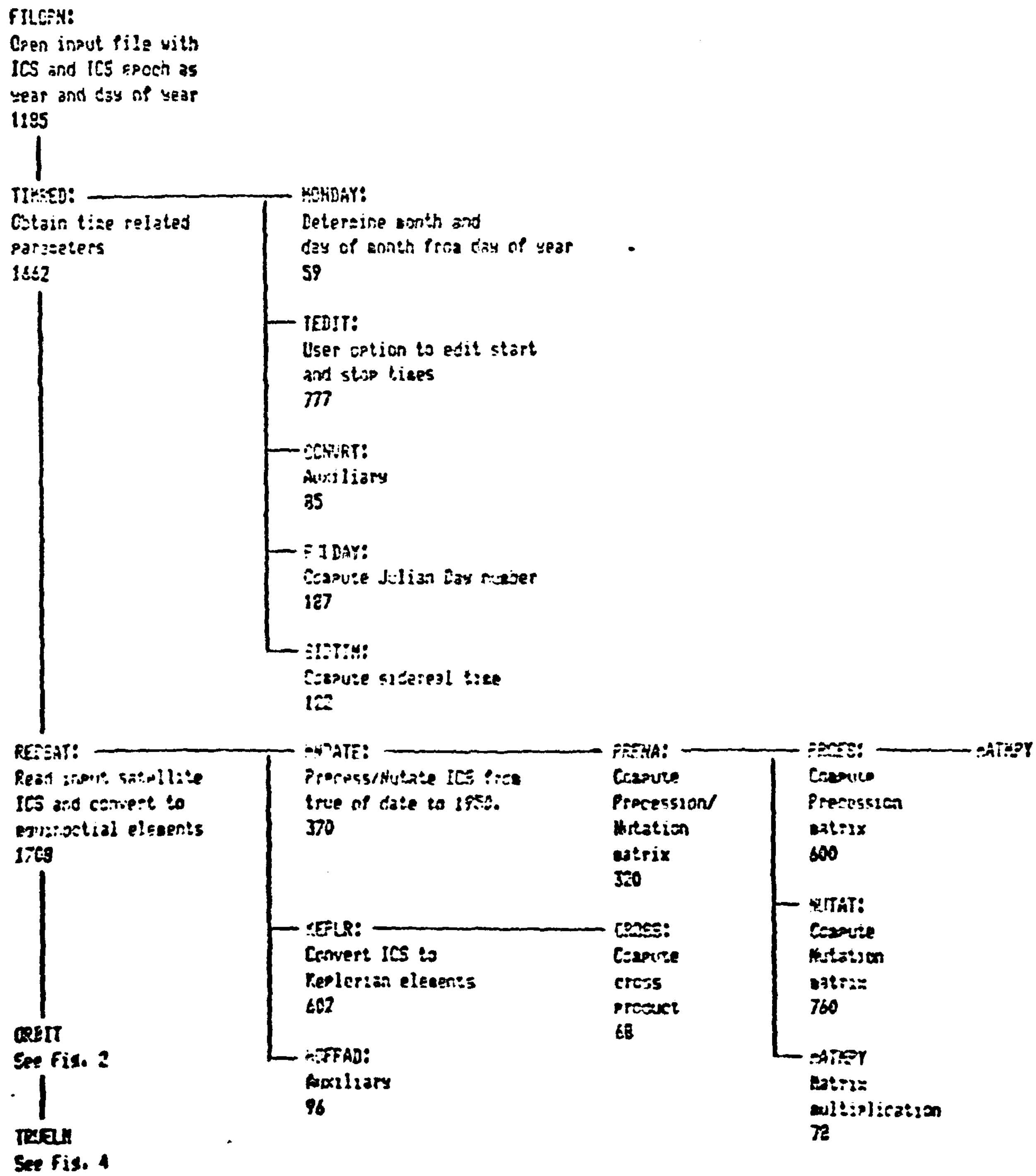

Fig. 1 Subroutine Structure Section 1, with name, description and number of words.

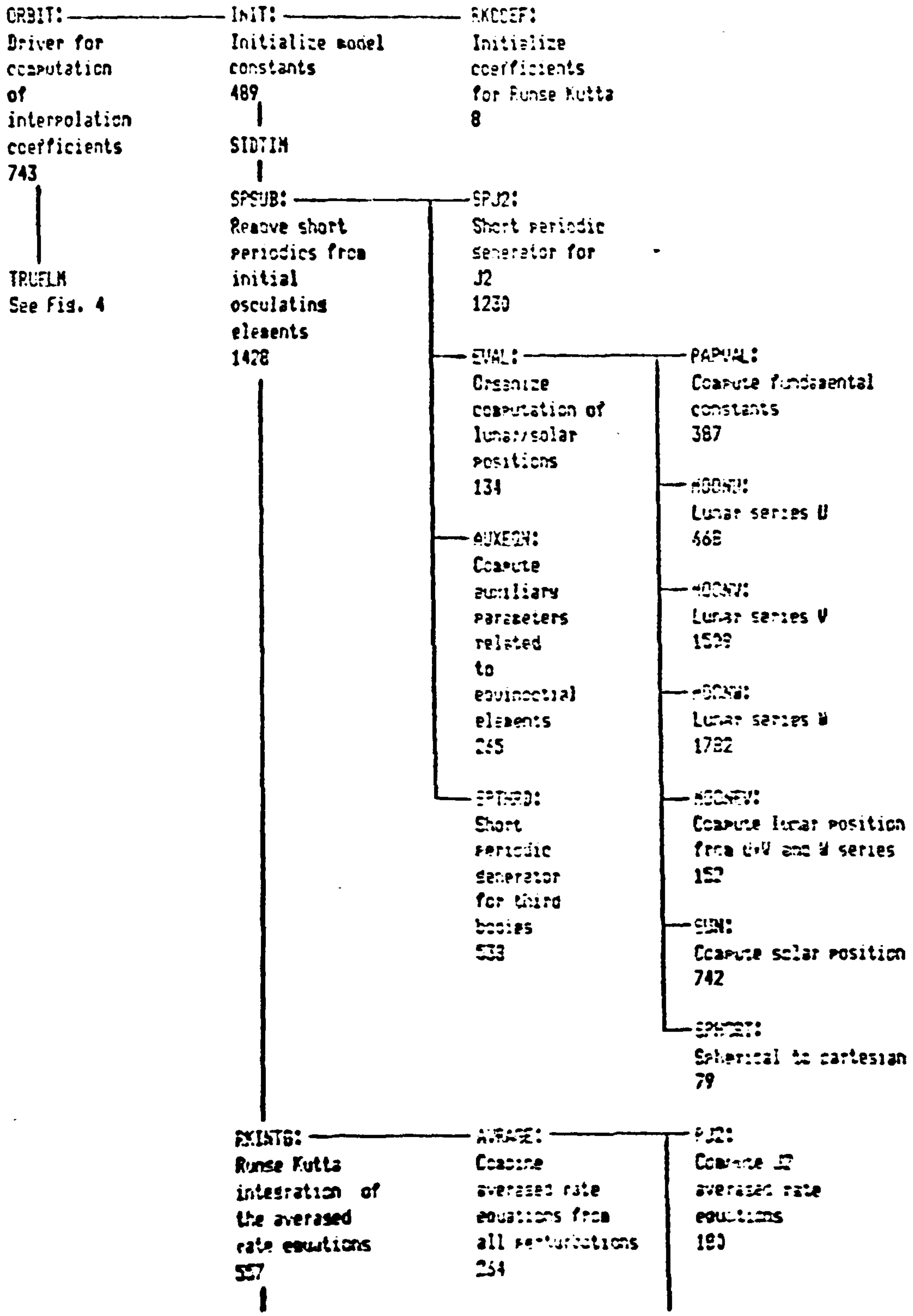

Fig. 2 Subroutine Structure Section 2, with name, description and number of words.

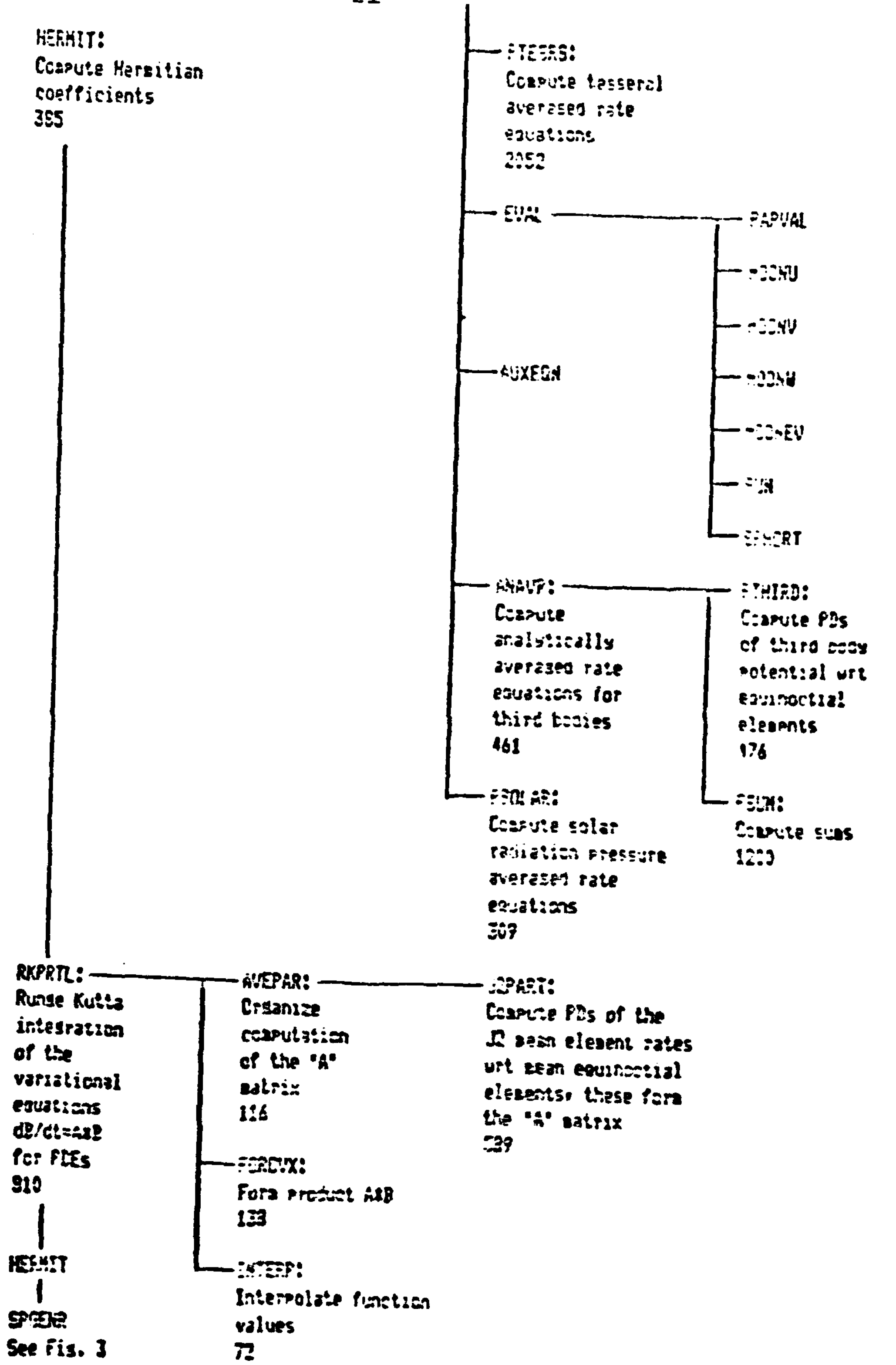

Fig. 2 Continued

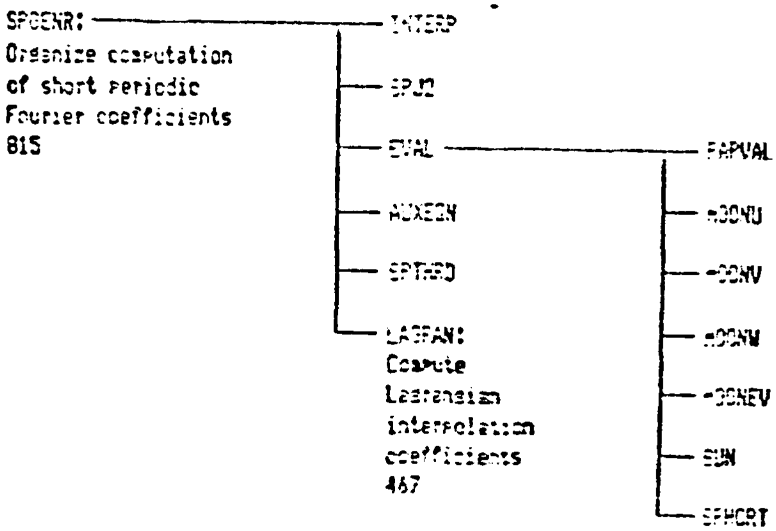

Fig. 3 Subroutine Structure Section 3, with name, description and number of words.

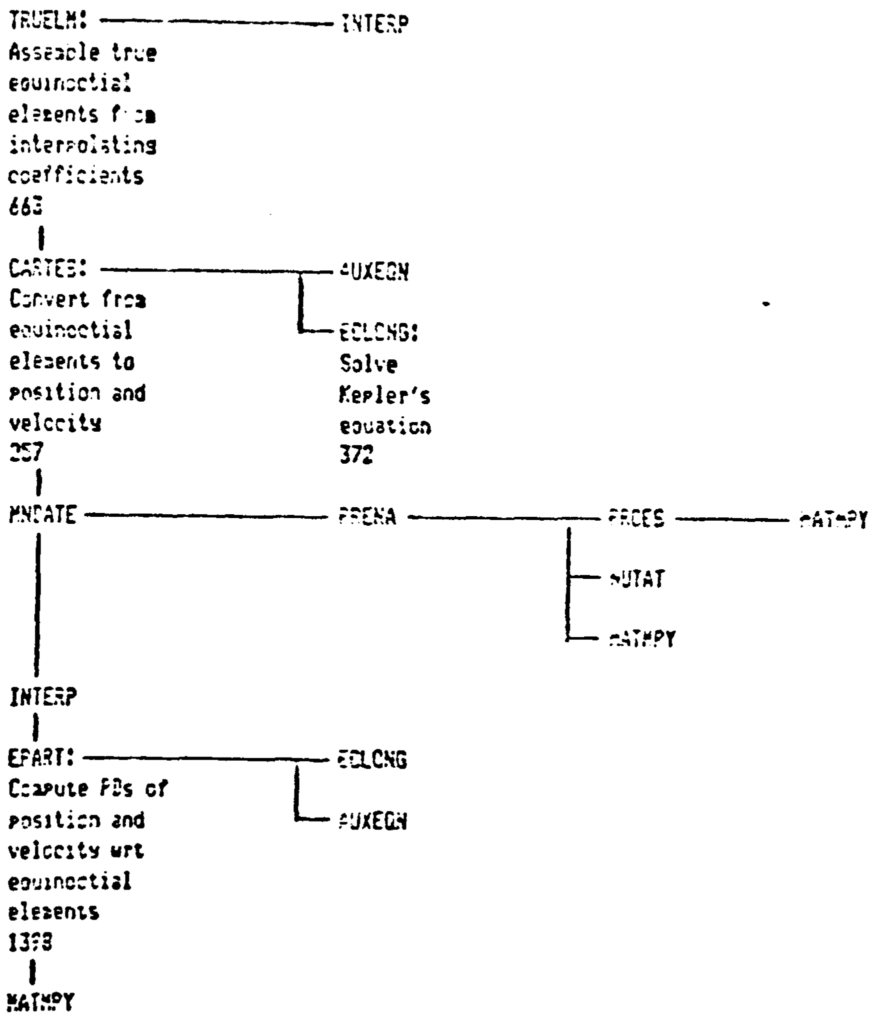

Fig. 4 Subroutine Structure Section 4, with name, description and number of words.

from a region can be resident in central memory at a time. Otherwise the return path to the main program would be destroyed. It is necessary though to make use of the fact that segments in one region can make calls to routines in segments of other regions. This is why three regions were established. Figure 5 shows how the overlay regions are constructed and interlinked.

In going to a small computer one expects a longer execution time for the program. The timing of the program's execution is separated into two parts: 1) the time required to set up all of the interpolation coefficients - ≈100 seconds; and 2) the time required to compute a satellite position from them - ≈1 second.

RESULTS

During the development of the LSI model two types of comparisons were made for debugging the program and for checking the error level

- comparison with the GTDS semianalytical model
- comparison with numerical integration.

Most of the comparisons with the GTDS model were for debugging purposes since the averaged rate equation and short periodic computations could be individually checked for each force model. The most significant comparison was made against a 7 day GTDS run where both models used the same initial mean elements. Agreement in position during the 7 day span was on the order of 20 meters.

For examining the error level of the LSI model, comparisons

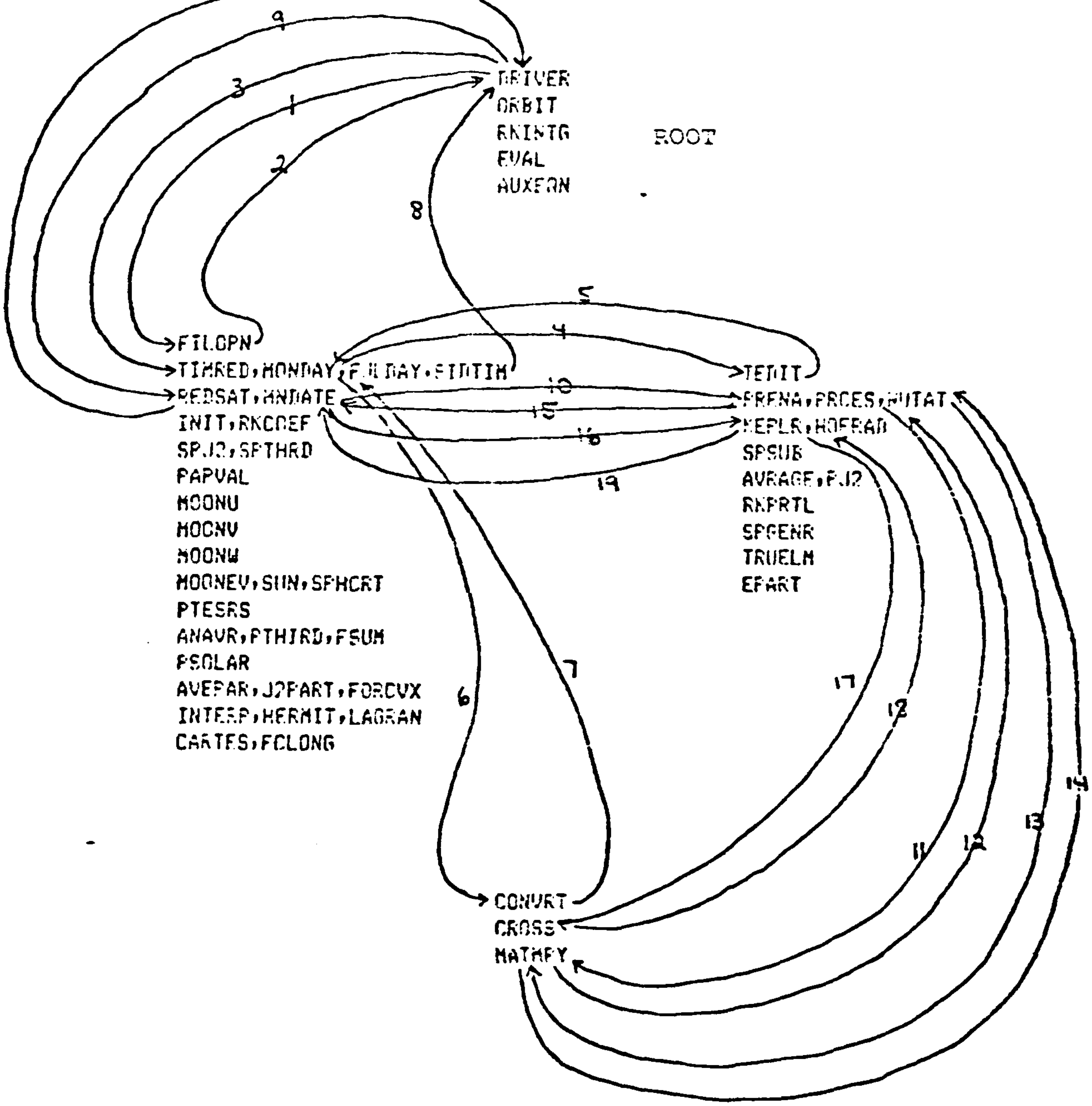

Fig. 5 The root and three overlay regions are shown. Each line of subroutines in a region defines a segment. The program pathway through the subroutine structure of Figure 1 is used to show an example of how segments are called from their regions and how the regions are interlinked.

were mostly made against numerical integration. Two possible numerical integration programs existed for the purpose of comparison - the GTDS system already mentioned and PEP (Ref. 18) which is used at MIT and is, as GTDS, a full blown model. As transfer of comparison files from the PEP computer program to the LSI was more convenient than from the AMDAHL, PEP was used.

Given a set of high precision orbital elements at epoch, a corresponding set of initial mean elements must be determined for the semianalytical model. It is important to obtain these mean elements accurately since errors in them lead to a secularly increasing prediction error. A least squares differential correction algorithm is used that solves for the epoch mean elements based on the fit of a semianalytical trajectory to the ephemeris output of a high precision Cowell integration. The *a priori* estimates of the mean elements at epoch are obtained from a single point conversion method. Here the short periodics of the model are computed at the epoch and subtracted from the osculating elements. PEP was used to generate the high precision ephemeris for a fit span of 36 hours with a 22.5 minute spacing.

Using the mean elements thus obtained, a semianalytical trajectory was generated again - this time for comparison purposes with the PEP ephemeris. Figs. 6, 7 and 8 show respectively radial, along track and cross track comparisons with PEP over the 36 hour fit span. Figs. 9, 10 and 11 show comparisons over the 36 hour fit span and the 36 hours beyond the fit span. The latter three figures show an expanding envelope of the residuals with time. This is due to various model differences. It appears that the error level of the LSI model is on the order of 2 ppm.

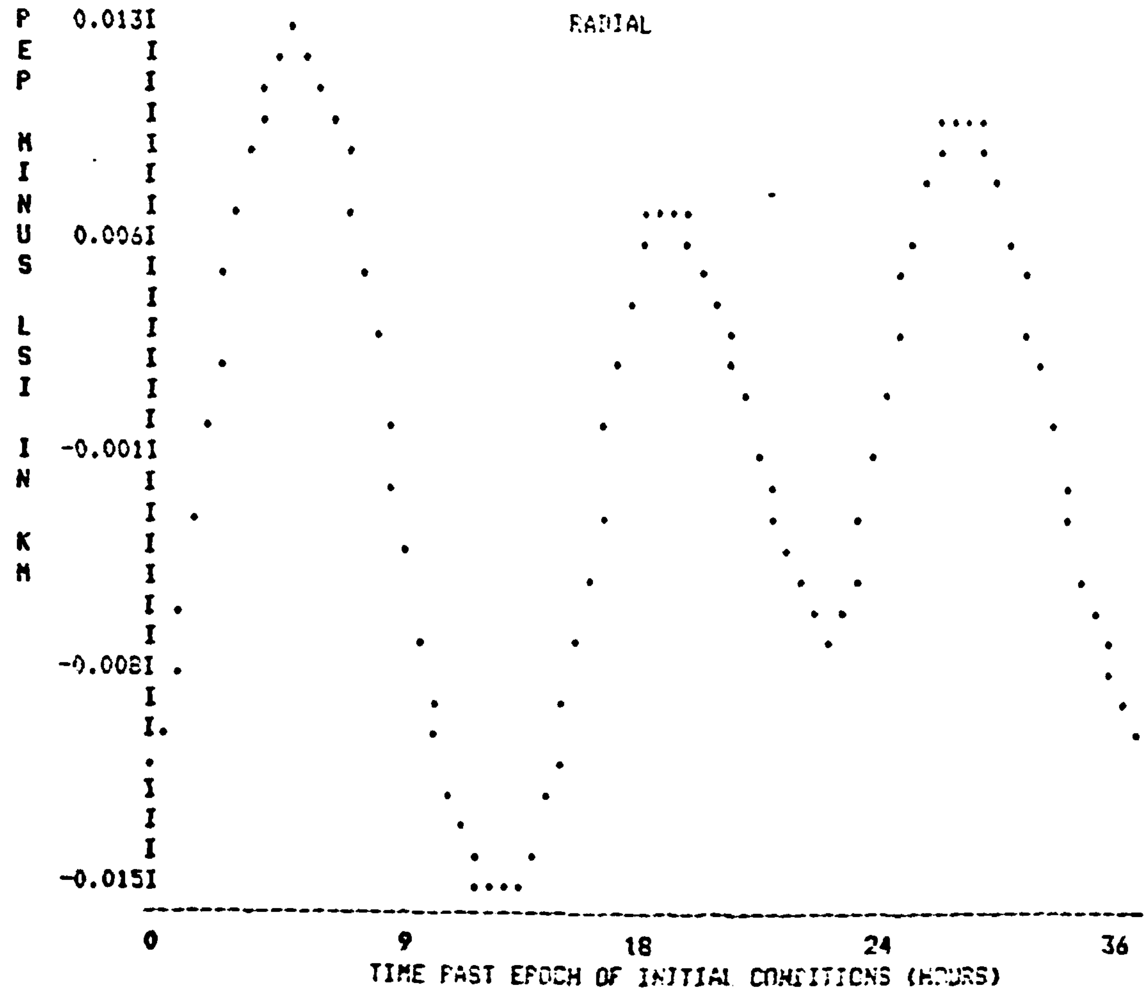

Fig. 6 LSI vs PEP in radial over the 36 hour fit span.

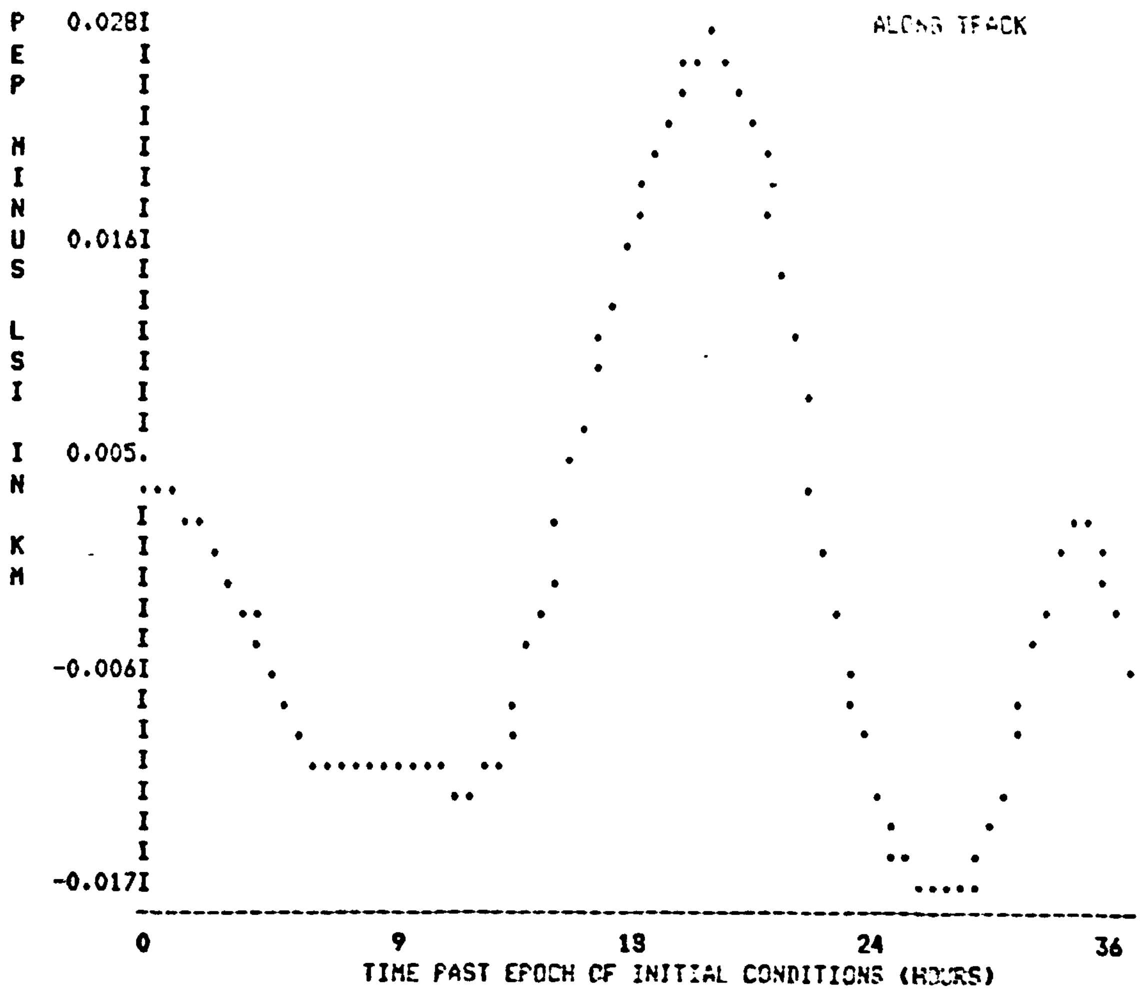

Fig. 7 LSI vs PEP in along track over the 36 hour fit span.

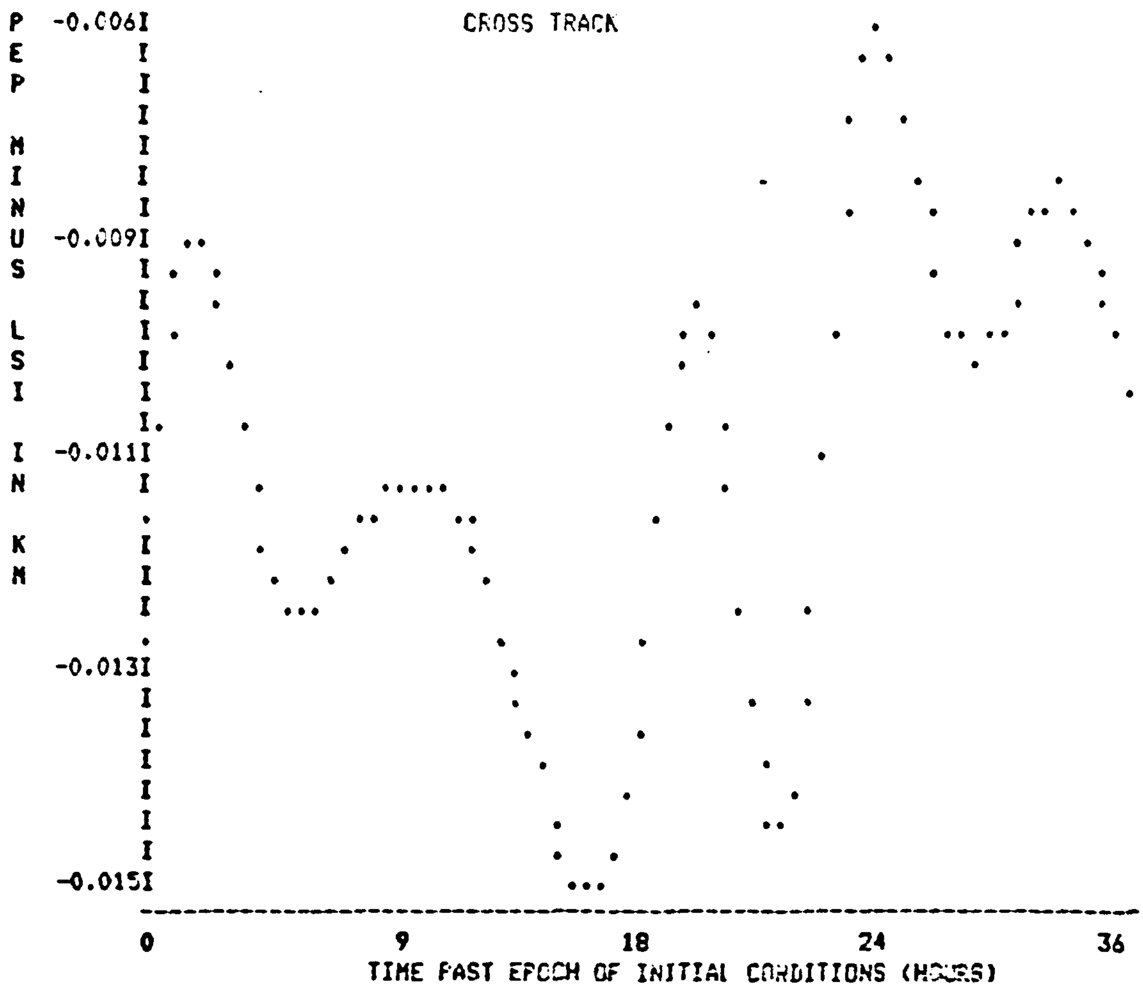

Fig. 8 LSI vs PEP in cross track over the 36 hour fit span.

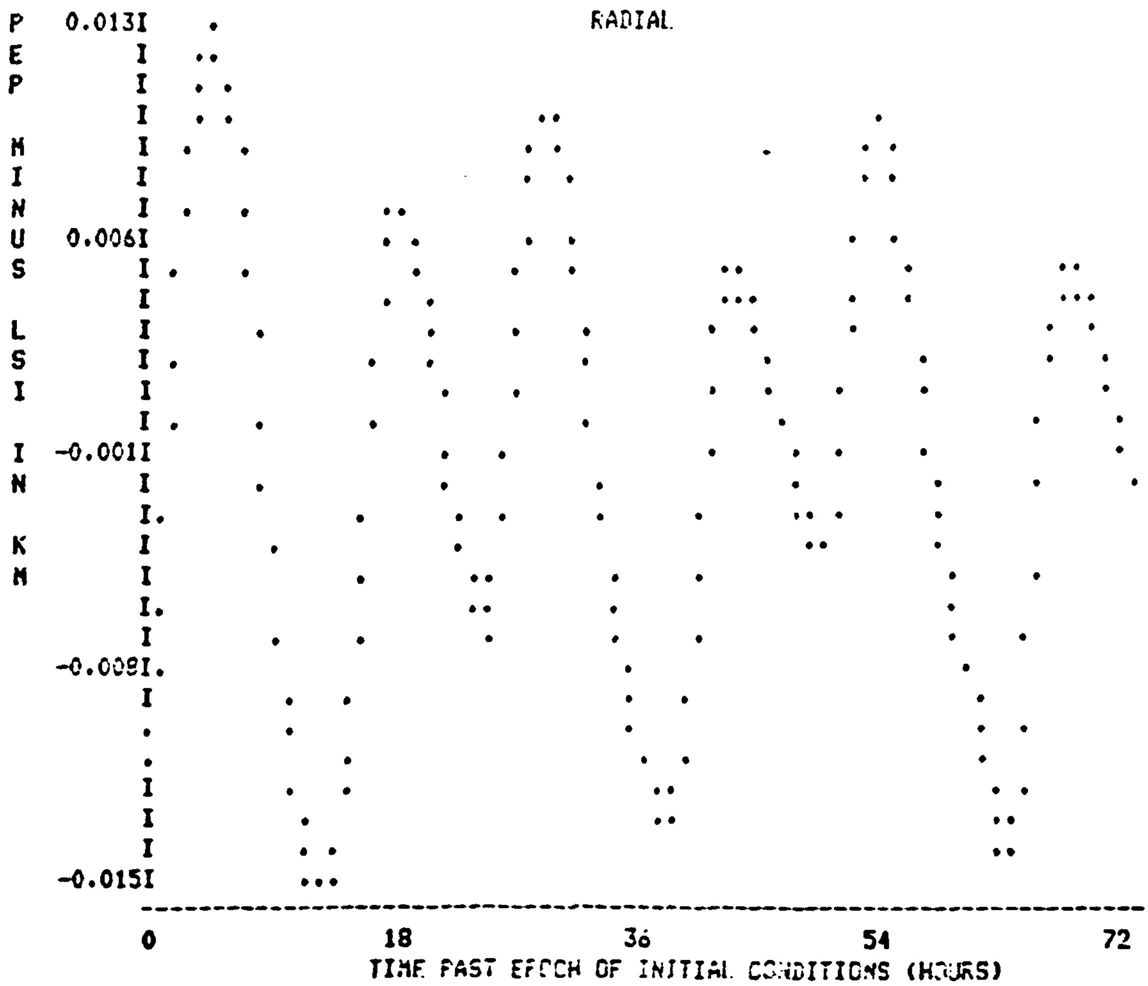

Fig. 9 LSI vs PEP in radial, the first 36 hours are over the fit span and the last 36 hours are over the predict span.

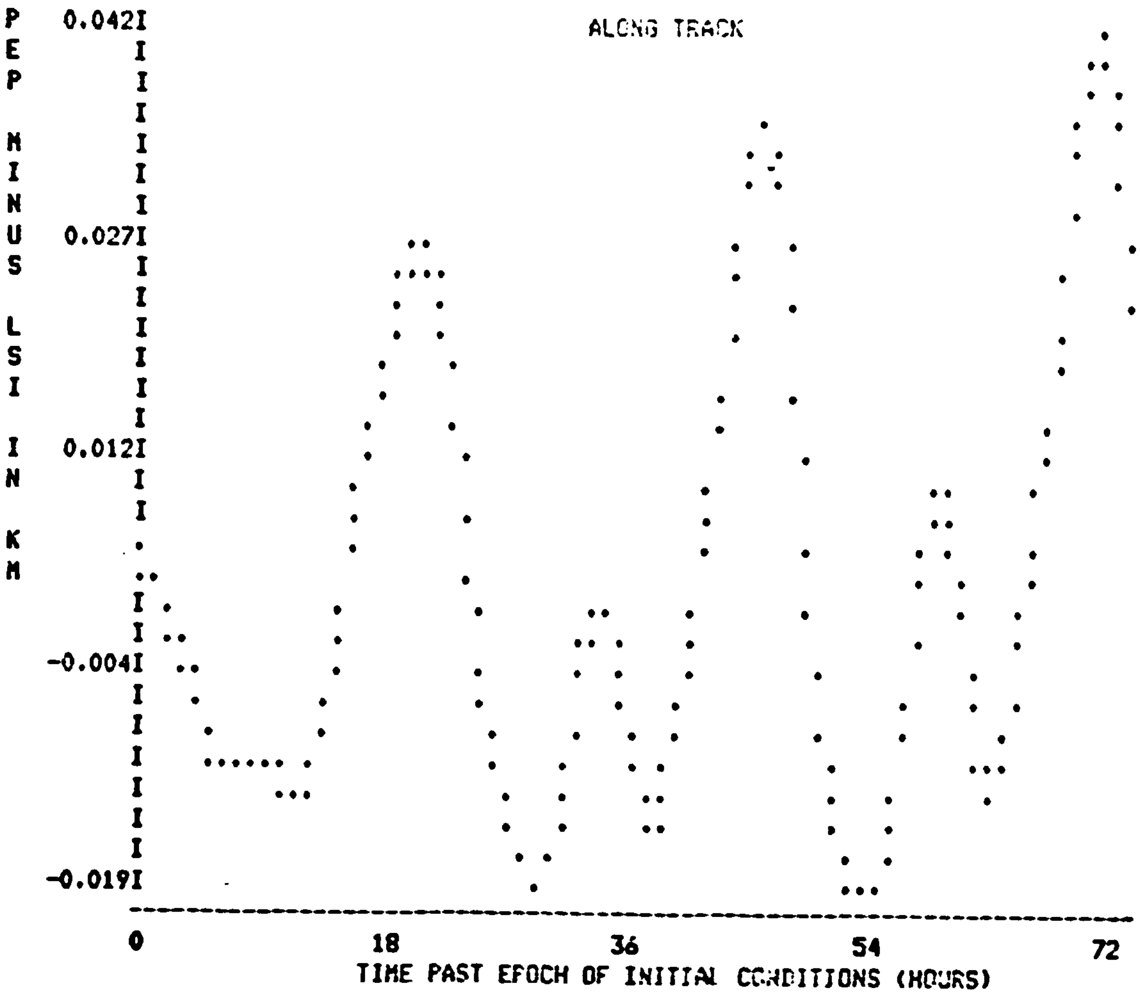

Fig. 10 LSI vs PEP in along track, the first 36 hours are over the fit span and the last 36 hours are over the predict span.

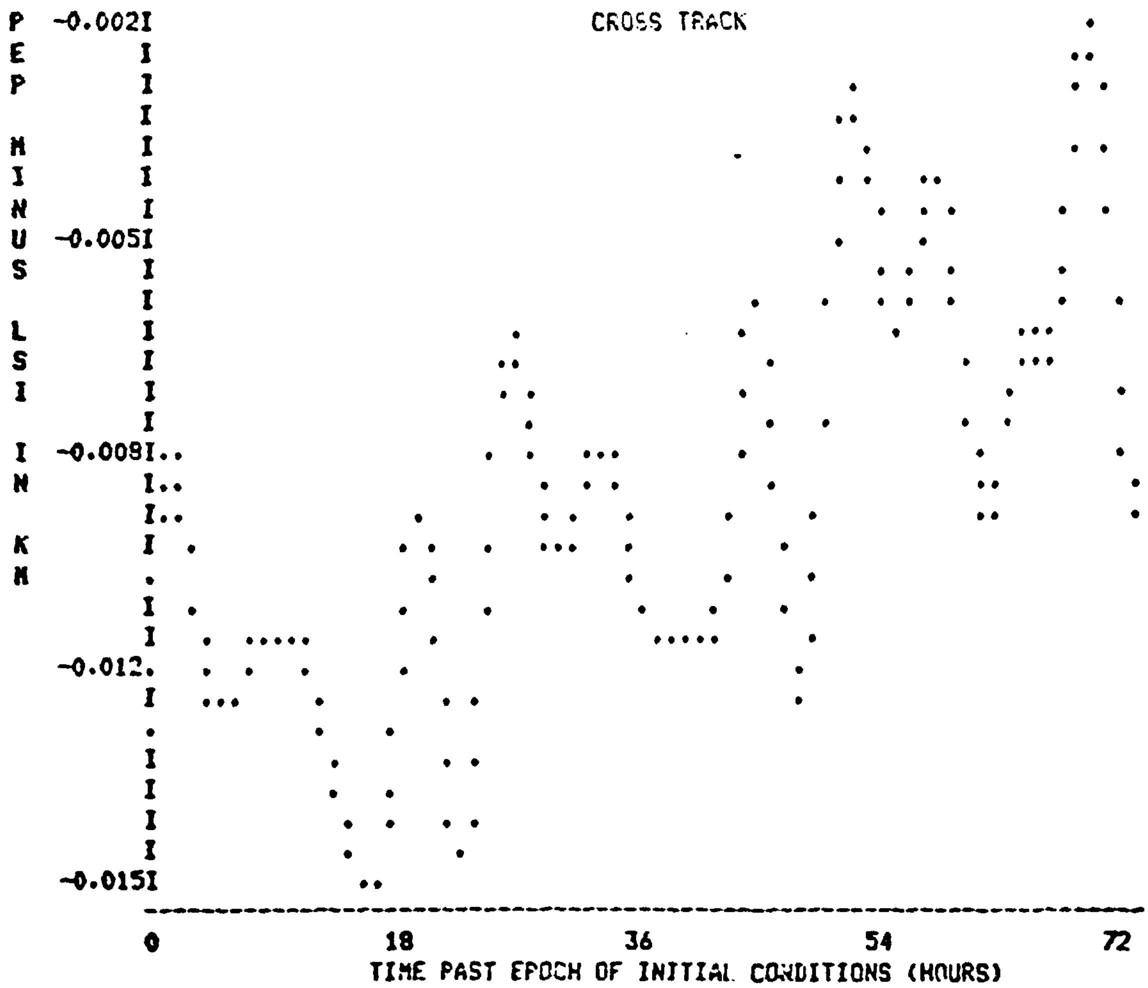

Fig. 11 LSI vs PEP in cross track, the first 36 hours are over the fit span and the last 36 hours are over the predict span.

FUTURE WORK

With the present overlay structure we have the flexibility of improving the accuracy of the model by incorporating some of the force models we have so far omitted. We would eventually like to handle solar radiation pressure in a more complete manner since this is an important effect for the high altitude GPS satellites. The program can be quite easily suited to synchronous satellites by modifying the subroutine modules for tesseral resonant perturbations and the lunar-solar short-periodics. Finally, we have immediate plans to fit real observations with the LSI model in order to determine initial mean elements.

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